Mr. Michael T. Davis WLPCS Calculus	Section 2.4 Deriving Slope Functions November 20, 2017

Name: Solutions / Answers

1. Use the limit as h approaches zero and the slope of a secant line to derive the slope function for the given function $f(x) = 2x^2 + 5$ at the point $x = a$. Show all the steps in the process very neatly and carefully. Do not rush through this.

Secant slope

$$m = \frac{f(a+h) - f(a)}{a+h-a} = \frac{f(a+h) - f(a)}{h}$$

Tangent line slope function

$$m = \lim_{h \to 0} \frac{2(a+h)^2 + 5 - (2a^2 + 5)}{h}$$

$$m = \lim_{h \to 0} \frac{2(a^2 + 2ah + h^2) + 5 - 2a^2 - 5}{h}$$

$$m = \lim_{h \to 0} \frac{\cancel{2a^2} + 4ah + 2h^2 \cancel{+5} - \cancel{2a^2} \cancel{-5}}{h}$$

$$m = \lim_{h \to 0} \frac{4ah + 2h^2}{h}$$

$$m = \lim_{h \to 0} (4a + 2h)$$

$m = 4a$ The slope function

2. Use the limit as h approaches zero and the slope of a secant line to derive the slope function for the given function $f(x) = 3x^2 - 7x$ at the point $x = a$. Show all the steps in the process very neatly and carefully. Do not rush through this.

Secant Slope

$$m = \frac{f(a+h) - f(a)}{a+h-a} = \frac{f(a+h) - f(a)}{h}$$

$$m = \frac{3(a+h)^2 - 7(a+h) - (3a^2 - 7a)}{h}$$

Tangent line slope function

$$m = \lim_{h \to 0} \frac{3(a+h)^2 - 7(a+h) - (3a^2 - 7a)}{h}$$

$$m = \lim_{h \to 0} \frac{3(a^2 + 2ah + h^2) - 7a - 7h - 3a^2 + 7a}{h}$$

$$m = \lim_{h \to 0} \frac{\cancel{3a^2} + 6ah + 3h^2 \cancel{-7a} - 7h \cancel{-3a^2} \cancel{+7a}}{h}$$

$$m = \lim_{h \to 0} \frac{6ah + 3h^2 - 7h}{h}$$

$$m = \lim_{h \to 0} (6a + 3h - 7)$$

$m = 6a - 7$ the slope function

3. Use the limit as h approaches zero and the slope of a secant line to derive the slope function for the given function $f(x)=x^3-4x^2$ at the point $x=a$. Show all the steps in the process very neatly and carefully. Do not rush through this.

$$m=\lim_{h\to 0}\frac{(a+h)^3-4(a+h)^2-(a^3-4a^2)}{h}$$

$$m=\lim_{h\to 0}\frac{\cancel{a^3}+3a^2h+3ah^2+h^3-4(a^2+2ah+h^2)-\cancel{a^3}+4a^2}{h}$$

$$m=\lim_{h\to 0}\frac{3a^2h+3ah^2+h^3-\cancel{4a^2}-8ah-4h^2+\cancel{4a^2}}{h}$$

$$m=\lim_{h\to 0}(3a^2+3ah+h^2-8a-4h)$$

$m=3a^2-8a$ The slope function

Mr. Michael T. Davis WLPCS Calculus	Section 2.4 Deriving Slope Functions (2) November 21, 2017

Name: Solutions / Answers

1. Use the limit as h approaches zero and the slope of a secant line to derive the slope function for the given function $f(x) = 2 + 8x - 3x^2$ at the point $x = a$. Show all the steps in the process very neatly and carefully. Do not rush through this.

$$m = \lim_{h \to 0} \frac{2 + 8(a+h) - 3(a+h)^2 - (2 + 8a - 3a^2)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{2} + \cancel{8a} + 8h - \cancel{3a^2} - 6ah - 3h^2 - \cancel{2} - \cancel{8a} + \cancel{3a^2}}{h}$$

$$= \lim_{h \to 0} \frac{8h - 6ah - 3h^2}{h}$$

$$= \lim_{h \to 0} (8 - 6a - 3h)$$

$= 8 - 6a$ the slope function

2. Use the limit as h approaches zero and the slope of a secant line to derive the slope function for the given function $f(x) = \sqrt{x}$ at the point $x = a$. Show all the steps in the process very neatly and carefully. Do not rush through this.

$$m = \lim_{h \to 0} \frac{\sqrt{a+h} - \sqrt{a}}{h}$$

$$= \lim_{h \to 0} \left(\frac{\sqrt{a+h} - \sqrt{a}}{h}\right)\left(\frac{\sqrt{a+h} + \sqrt{a}}{\sqrt{a+h} + \sqrt{a}}\right)$$

$$= \lim_{h \to 0} \frac{\cancel{a}+h + \cancel{\sqrt{a^2+ah}} - \cancel{\sqrt{a^2+ah}} - \cancel{a}}{h\left(\sqrt{a+h} + \sqrt{a}\right)}$$

$$= \lim_{h \to 0} \frac{h}{h\left(\sqrt{a+h} + \sqrt{a}\right)}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{a+h} + \sqrt{a}}$$

$$= \frac{1}{\sqrt{a} + \sqrt{a}}$$

$$= \frac{1}{2\sqrt{a}} \quad \text{slope function}$$

3. Use the limit as h approaches zero and the slope of a secant line to derive the slope function for the given function $f(x)=\frac{1}{x}$ at the point $x=a$. Show all the steps in the process very neatly and carefully. Do not rush through this.

$$m = \lim_{h\to 0} \frac{\frac{1}{a+h} - \frac{1}{a}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{a}{a(a+h)} - \frac{a+h}{a(a+h)}}{h}$$

$$= \lim_{h\to 0} \left(\frac{\cancel{a} - \cancel{a} - h}{a(a+h)} \cdot \frac{1}{h}\right)$$

$$= \lim_{h\to 0} \left(\frac{-h}{a(a+h)} \cdot \frac{1}{h}\right)$$

$$= \lim_{h\to 0} \frac{-1}{a(a+h)}$$

$$= \frac{-1}{a(a)}$$

$$= \frac{-1}{a^2} \quad \text{slope function}$$